



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

- 1 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} - 8\frac{dx}{dt} - 9x = 9e^{8t}. \quad [6]$$

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2 Let $I_n = \int_0^1 (1 + 3x)^n e^{-3x} dx$, where n is an integer.

(a) Show that $3I_n = 1 - 4^n e^{-3} + 3nI_{n-1}$. [3]

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(b) Find the exact value of I_2 . [3]

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3 The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 7 \\ 0 & 6 & 0 \\ 7 & 7 & 5 \end{pmatrix}.$$

(a) Find the eigenvalues of \mathbf{A} .

[4]

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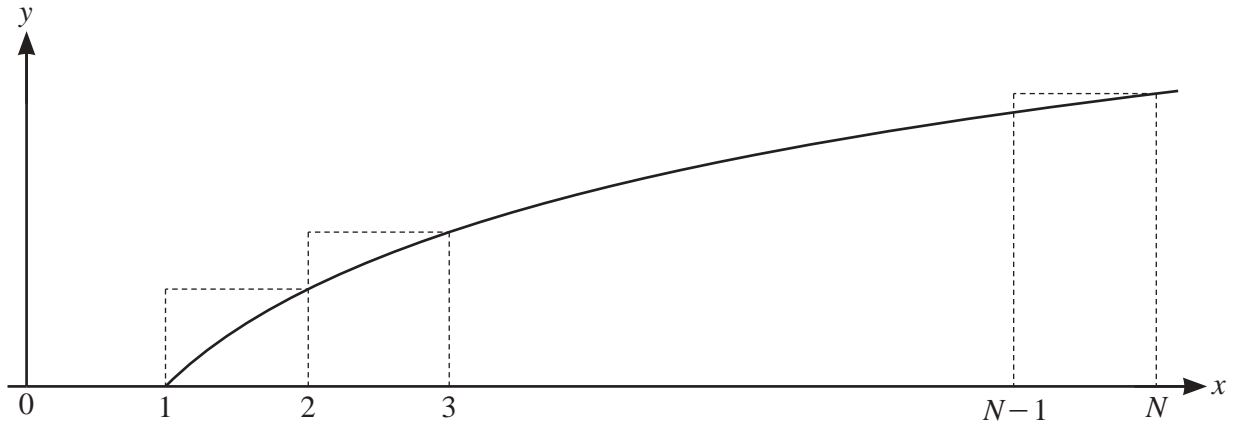
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The diagram shows the curve with equation $y = \ln x$ for $x \geq 1$, together with a set of $(N - 1)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\ln N! > N \ln N - N + 1. \quad [5]$$

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5 The curve C has parametric equations

$$x = \frac{1}{2}t^2 - \ln t, \quad y = 2t + 1, \quad \text{for } \frac{1}{2} \leq t \leq 2.$$

(a) Find the exact length of C .

[5]

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(b) Find $\frac{d^2y}{dx^2}$ in terms of t , simplifying your answer.

[4]

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6 (a) Starting from the definitions of \tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2 \theta = \operatorname{sech}^2 \theta. \quad [3]$$

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The variables x and y are such that $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$, for $-\frac{1}{4}\pi < x < \frac{3}{4}\pi$.

(b) By differentiating the equation $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$ with respect to x , show that

$$\frac{dy}{dx} = -\operatorname{cosec}\left(x + \frac{1}{4}\pi\right). \quad [4]$$

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(c) Hence find the first three terms in the Maclaurin’s series for $\tanh^{-1}\left(\cos\left(x + \frac{1}{4}\pi\right)\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants a , b and c . [5]

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(b) Hence find the solution of the differential equation

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

for which $y = \ln 2$ when $x = 0$. Give your answer in the form $y = f(x)$. [7]

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- 8 (a)** Use de Moivre's theorem to show that $\sin^6 \theta = -\frac{1}{32}(\cos 6\theta - 6 \cos 4\theta + 15 \cos 2\theta - 10)$. [6]

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It is given that $\cos^6 \theta = \frac{1}{32}(\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10)$.

- (b)** Find the exact value of $\int_0^{\frac{1}{3}\pi} (\cos^6(\frac{1}{4}x) + \sin^6(\frac{1}{4}x)) dx$. [4]

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(c) Express each root of the equation $16c^6 + 16(1 - c^2)^3 - 13 = 0$ in the form $\cos k\pi$, where k is a rational number. [5]

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Additional Page

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